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ANALYSIS OF ERRORS INTRODUCED BY SEVERAL METHODS OF WEIGHTING NONUNIFORM DUCT FLOWS

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SUMMARY

Various weighting methods are applied to typical nonuniform duct flow profiles to determine average flow properties. The analysis covers a range of subsonic duct Mach numbers, but is confined to flows having uniform static pressure and total temperature.

An averaging method is developed which yields uniform properties that reproduce the mass and momentum of the nonuniform flow. In contrast, it is shown that the use of conventional weighting methods may result in large errors in these properties. These errors are shown to have varying significance depending on the applications to which the data are applied.

It is also shown that nonuniform flows through variable-area duct passages will cause changes in average flow properties that are not associated with the real thermodynamic flow path.

INTRODUCTION

In most calculations involving duct air-flow properties, it is not convenient to consider local flow variations within the duct. Therefore, the properties of the flow are treated as though they were uniformly distributed, and one-dimensional equations are applied to this uniform flow. Inasmuch as the real flow seldom approaches uniformity at planes of interest, the equivalent uniform flow must be determined by some method of averaging the properties of the real flow.

This report presents the results of an analytical study made to determine the accuracy with which several commonly used averaging or weighting methods reproduce the real flow properties. The significance of inherent errors is illustrated for several common applications of duct flow data. Errors introduced through the application of one-dimensional relations to the uniform flow are briefly examined.

The study considers several typical velocity gradients but is confined to subsonic compressible flows with uniform static pressures and stagnation temperatures.

(Since the present analysis was completed, it has been found that a more generalized, qualitative analysis of the same problem is contained in ref. 1.)

ANALYSIS

A uniform flow representing the flow properties of a nonuniform duct flow should satisfy the total energy, mass, and momentum of the real flow. For the special case considered herein in which the flow is assumed to arise from a uniform temperature source and to flow adiabatically to the measuring station, the total energy of the real flow can be reproduced by the assumption of constant total temperature in the uniform flow at the source value. The determination of a uniform flow that will simultaneously satisfy the mass flow and the momentum in the real flow is more difficult.

Mass-Momentum Method

For the special case in which the static pressure and total temperature are constant across the duct, the mass flow is given by the equation

$$m = \sqrt{\frac{\gamma}{RT}}\, p \int M\left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{1}{2}} dA \tag{1}$$

where M is the axial component of the local duct Mach number. (All symbols are defined in appendix A.)

In order for the mass flow in the representation to equal this integrated mass flow, the uniform flow must satisfy the relation

$$m = \sqrt{\frac{\gamma}{RT}}\, p_e M_e \left(1 + \frac{\gamma-1}{2} M_e^2\right)^{\frac{1}{2}} A \tag{2}$$

where p_e and M_e are the effective static pressure and Mach number, respectively.

The integrated momentum of the real flow can be expressed by

$$\varphi = p \int (1 + \gamma M^2) dA \tag{3}$$

Thus, the effective static pressure and Mach number must also satisfy the relation

$$\varphi = p_e(1 + \gamma M_e^2)A \tag{4}$$

By combining equations (1) to (4), the expression for the effective Mach number required to satisfy the total energy, mass flow, and momentum of the real flow becomes

$$\frac{m\sqrt{\frac{RT}{\gamma}}}{\varphi} = \frac{M_e\left(1 + \frac{\gamma-1}{2} M_e^2\right)^{\frac{1}{2}}}{1 + \gamma M_e^2} \tag{5}$$

where m and φ are integrated values determined from equations (1) and (3).

Although for this analysis the static pressure is assumed to be constant across the real duct flow, this measured value of pressure cannot be used in conjunction with the effective Mach number determined from equation (5) to satisfy the real flow properties. Instead, a new effective static pressure must be determined from either the momentum or the mass-flow equations as

$$p_e = p\,\frac{\int(1 + \gamma M^2)dA}{(1 + \gamma M_e^2)A} = p\,\frac{\int M\left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{1}{2}} dA}{M_e\left(1 + \frac{\gamma-1}{2} M_e^2\right)^{\frac{1}{2}} A} \tag{6}$$

This effective static pressure is never identical to the measured pressure if velocity gradients are present in the real flow.

To complete the definition of the equivalent uniform flow, an effective total pressure can be determined from the expression

$$P_e = p_e\left(1 + \frac{\gamma-1}{2} M_e^2\right)^{\frac{\gamma}{\gamma-1}} \tag{7}$$

The flow quantities defined by this method of averaging would be those obtained by mixing the measured profile to a uniform flow in a constant-area duct without wall friction. Mixing losses are inherently contained in the average flow quantities.

Conventional Weighting Methods

The weighting or averaging methods commonly used to obtain uniform flow representations of nonuniform duct flows require either less complicated data-collection methods or less tedious calculation techniques than does the exact weighting procedure. Such methods result in inherent errors in the representation of one or more of the basic properties of the real flow. The required assumptions and applicable equations for three of the more commonly utilized methods follow.

Mass-derived method. - When the mass flow in the duct is known from some independent measurement, the measured static pressure at a station can be used in conjunction with the geometrical flow area A to define a uniform duct Mach number M_c that satisfies the mass flow by the equation

$$M_c\left(1 + \frac{\gamma-1}{2} M_c^2\right)^{\frac{1}{2}} = \frac{m\sqrt{\frac{RT}{\gamma}}}{pA} \tag{8}$$

From this average Mach number and the measured static pressure, an average total pressure P_c can be calculated as

$$P_c = p\left(1 + \frac{\gamma-1}{2} M_c^2\right)^{\frac{\gamma}{\gamma-1}} \tag{9}$$

The momentum calculated from the measured static pressure and the average Mach number becomes

$$\varphi_c = p(1 + \gamma M_c^2)A \tag{10}$$

It is evident that the mass flow and total energy of the real duct flow are inherently satisfied by the mass-derived method of determining an average flow. There is no attempt in this method, however, to satisfy the momentum of the real flow.

Mass-flow-weighting method. - A pitot-static survey of the flow at the desired duct station is frequently employed to determine an average uniform flow. If it is assumed that the measured nonuniform flow can be brought to rest without mixing losses, the resultant pressure can be determined from the equation

$$P_c = \frac{\int P\ dm}{\int dm} = \frac{\int P\rho V\ dA}{\int \rho V\ dA} \tag{11a}$$

For the special case in which the static pressure and total temperature are constant across the duct, the compressible form of equation (11a) becomes

$$P_c = \frac{p\int M\left(1 + \frac{\gamma-1}{2} M^2\right)^4 dA}{\int M\left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{1}{2}} dA} \tag{11b}$$

The mass flow and momentum of the uniform flow having a total pressure defined by equations (11) are not unique values. Their magnitudes depend upon the nature of additional assumptions about the properties of the uniform flow.

The measured static pressure at the duct station is often assumed to be the static pressure of the average flow. With this assumption, a uniform duct Mach number can be defined by the relation

$$M_c = \left\{\frac{2}{\gamma-1}\left[\left(\frac{P_c}{p}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]\right\}^{\frac{1}{2}} \tag{12}$$

The momentum for this uniform flow is given by equation (10). The calculated mass flow becomes

$$m_c = \sqrt{\frac{\gamma}{RT}}\, pAM_c\left(1 + \frac{\gamma-1}{2} M_c^2\right)^{\frac{1}{2}} \tag{13}$$

The mass flow determined from equation (13) will not correspond to the integrated mass flow which was used to determine the average total pressure in equation (11a).

This anomaly between the integrated and calculated mass flows can be avoided by defining an average static pressure p_c which, when used with the average total pressure from equations (11), will satisfy the integrated mass flow. The average Mach number required to satisfy the mass flow under these conditions is given by

$$\frac{M_c}{\left(1 + \frac{\gamma-1}{2} M_c^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}} = \frac{m\sqrt{\frac{RT}{\gamma}}}{P_c A} \tag{14}$$

and the resultant static pressure becomes

$$p_c = \frac{P_c}{\left(1 + \frac{\gamma-1}{2} M_c^2\right)^{\frac{\gamma}{\gamma-1}}} \tag{15}$$

The momentum calculated from equation (10) with either the measured or calculated values of static pressure and corresponding Mach number will not equal the integrated momentum. Equations similar to (14) and (15) can be determined which would yield a static pressure and Mach number for the uniform flow that would satisfy the real flow momentum. These flow properties would not satisfy the mass flow, however, and are not conventionally employed.

Area-weighting method. - When pitot-static flow surveys are employed, the complications of the calculation procedure can be reduced by using an area-weighted average total pressure determined from the equation

$$P_c = \frac{\int P \; dA}{A} = \frac{p\int\left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{7}{2}} dA}{A} \tag{16}$$

The remaining properties of the uniform flow are calculated by the equations used with the mass-flow-weighting method. As in the former method, several solutions for these properties are possible. Generally, the static pressure is assumed equal to the measured value. If independent mass measurements are available, a static pressure may be calculated to satisfy the mass flow. With compressible duct flow, the integrated momentum will not be satisfied with either assumption. (For the incompressible case, a uniform flow defined by the total pressure from eq. (16) and the measured static pressure will duplicate the real flow momentum.)

NUMERICAL CALCULATIONS

The uniform flow properties of three arbitrary duct profiles were calculated by the mass-momentum weighting procedure and by the conventional weighting methods discussed in ANALYSIS. For simplicity, the ducts were assumed square with symmetrical two-dimensional profiles. The profiles considered were:

(a) A power profile described by

$$M = Kx^{\frac{1}{7}} \tag{17}$$

(b) A discontinuous, separation profile represented by

$$0 < x < 0.1 \qquad M = 0 \tag{18a}$$

$$0.1 < x < 1.0 \qquad M = K \tag{18b}$$

(c) A linear profile of the form

$$M = K(0.2x + 0.8) \tag{19}$$

Each profile was evaluated for a range of values of K (corresponding to the maximum Mach number at the duct centerline) from 0 to 1.0.

Mass-momentum method. - Equation (1) was integrated for each of the profiles to determine the mass flow actually contained in the duct. The integrals for the power and linear profiles were approximated by series expansion. The resultant expressions for the mass flow (valid for $K < 1.0$) were

$$\frac{m\sqrt{\frac{RT}{\gamma}}}{pA} = 0.875K + 0.070K^3 - 0.00292K^5 + 0.00025K^7 - \ldots \text{ (Power profile)} \tag{20a}$$

$$= 0.9K\left(1 + \frac{K^2}{5}\right)^{\frac{1}{2}} \qquad \text{(Separation profile)} \tag{20b}$$

$$= 0.9K + 0.0738K^3 - 0.003074K^5 + 0.00026K^7 - \ldots \text{ (Linear profile)} \tag{20c}$$

The actual momentum with the assumed profiles was obtained by integrating equation (3) with the resultant expressions

$$\frac{\varphi}{pA} = 1 + 1.08889K^2 \qquad \text{(Power profile)} \tag{21a}$$

$$= 1 + 1.26K^2 \qquad \text{(Separation profile)} \tag{21b}$$

$$= 1 + 1.1387K^2 \qquad \text{(Linear profile)} \tag{21c}$$

Effective values of duct Mach number, static pressure, and total pressure were determined from equations (5), (6), and (7), respectively.

Mass-derived method. - By using the values of mass flow from equations (20), the properties of the uniform flow were determined from equations (8) to (10).

Mass-flow-weighting method. - The product of the average total pressure and the mass flow was obtained from equation (11b) and modified to the form

$$\frac{P_c}{p}\,\frac{m\sqrt{\frac{RT}{\gamma}}}{pA} = \int_0^1 M\left(1+\frac{\gamma-1}{2}M^2\right)^4 dx \tag{22}$$

Equation (22), when integrated, yielded the following expressions:

$$\frac{P_c}{p}\,\frac{m\sqrt{\frac{RT}{\gamma}}}{pA} = 0.875K + 0.56K^3 + 0.14K^5 + 0.016K^7 + 0.0007K^9 \quad \text{(Power profile)} \tag{23a}$$

$$= 0.9K\left(1+\frac{K^2}{5}\right)^4 \quad \text{(Separation profile)} \tag{23b}$$

$$= 0.9K + 0.5904K^3 + 0.1476K^5 + 0.01665K^7 + 0.00071K^9 \quad \text{(Linear profile)} \tag{23c}$$

The values of integrated mass flow from equations (20) were then used to obtain the average total pressure. Equations (12) to (15) were used, as appropriate, to determine the calculated average properties of the flow.

Area-weighting method. - The average total pressure was obtained from equation (16), which becomes

$$\frac{P_c}{p} = \int_0^1 \left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{7}{2}} dx \tag{24}$$

The resultant expressions, after integration, were

$$\frac{P_c}{p} = 1 + 0.5444K^2 + 0.1114K^4 + 0.00942K^6 + 0.000204K^8 - \ldots \quad \text{(Power profile)} \tag{25a}$$

$$= 0.1 + 0.9\left(1+\frac{K^2}{5}\right)^{\frac{7}{2}} \quad \text{(Separation profile)} \tag{25b}$$

$$= 1 + 0.5693K^2 + 0.1177K^4 + 0.00988K^6 + 0.00021K^8 - \ldots \quad \text{(Linear profile)} \tag{25c}$$

The average properties of the flow were calculated from equations (12) to (15).

RESULTS

The profiles assumed for the numerical analysis were chosen to represent conservative flow nonuniformities as compared with those often experimentally observed. Typical profiles are presented in figure 1. Each profile in this figure corresponds to an effective duct Mach number of 0.2 as determined by the mass-momentum method. In addition to the Mach number profiles, the accompanying total-pressure variations (for a constant duct static pressure) are presented in the form of the local incremental deviation in total pressure from the mean effective value determined by the mass-momentum method.

At the duct centerline $(x = 1.0)$, the maximum total-pressure deviation occurred with the power profile. In this case the local total pressure exceeded the effective average value by about 1 percent. With the wall pressure (at $x = 0$) used as an indication of the other extreme in total-pressure deviation, the separated profile gave a maximum deviation of less than $3\frac{1}{2}$ percent below the effective value. For purposes of qualitative comparison, the wall static pressure that would be observed for a uniform duct Mach number of 0.2 is indicated in the figure. It can be concluded, therefore, that all the assumed profiles represent moderate flow distortions. As a consequence, the errors that will be shown to accompany the various weighting techniques are less than might be expected for practical flow problems.

Figure 2 compares the static pressures that would be measured for each of the assumed profiles with the corresponding effective static pressures determined by the mass-momentum method. It is seen that the measured static pressure in a duct having nonuniform velocities will always be less than the effective static pressure required to describe the integrated flow properties in the duct.

The deviation between measured and effective static pressures increases as the effective duct Mach number increases for the assumed profiles. This results from the inherent nature of the profile assumptions, wherein the magnitude of the total-pressure variation across the duct increases as the maximum duct Mach number K, and hence the effective Mach number, increases. The curves terminate at a value of $K = 1$ for each profile. It is interesting to note that an effective duct Mach number of unity, as defined by the mass-momentum method, cannot be achieved with any nonuniform duct flow, regardless of the value of K. This restriction arises from the fact that the mass flow with uniform sonic velocity is greater than the mass flow in a nonuniform stream of the same area, whether subsonic or supersonic.

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The average total pressure determined by each of the weighting methods for the assumed profiles is compared with the effective value of total pressure from the mass-momentum method in figure 3. These total pressures are independent of any further assumptions regarding the average static pressure in the duct.

Mixing losses are inherently included in determining the effective total pressure by the mass-momentum method. Since the mass-flow-weighting method assumed no mixing losses, this method always yields an average total pressure that is greater than the effective value. On the other hand, the mass-derived and area-weighting methods yield average total pressures that are lower than the effective value. The errors with all methods of averaging increase as the severity of the profile increases whether through an increase in the value of K, and hence of M_c, or from the nature of the basic profile shape.

For weighting methods in which the uniform Mach number in the duct is determined from the calculated total pressure and the measured static pressure, the combined errors in static pressure (fig. 2) and calculated total pressure (fig. 3) might be expected to produce significant errors in Mach number. This expectation is confirmed by the curves of figure 4, which show that all the weighting methods yielded calculated Mach numbers that were greater than the corresponding effective Mach numbers determined by the mass-momentum method.

Any errors in the determination of static pressure and Mach number for the uniform flow will reflect as errors in the calculated mass flow and momentum. The magnitude of these errors is illustrated in figures 5 and 6 for the profiles and weighting methods considered.

Inasmuch as all the averaging methods used the measured static pressure in the calculation of mass flow in figure 5 and of momentum in figure 6, the ratio of measured to effective static pressure was identical. The differences in the calculated values therefore arise from the differences in Mach number computed by the various methods. With the mass-derived method, the calculated Mach number exactly satisfied the measured mass flow when used with the measured static pressure. The Mach number ratios indicated in figure 4 for the mass-derived method are, therefore, the ratios giving zero mass-flow error. For any given profile and effective duct Mach number, the Mach number ratios were higher for the area-weighting and mass-flow-weighting methods than for the mass-derived method, which explains the excessive mass flows computed by these methods.

In the case of the momentum computations, none of the averaging methods gives the base value of the Mach number ratio that is required to exactly compensate for the static-pressure error and reduce the momentum error to zero. It is possible to determine the necessary Mach number ratio, however, by equating equations (4) and (10). With the separation profile at an effective duct Mach number of 0.71 $(K = 1)$, for example, the ratio of measured to effective static pressure is 0.753 for all averaging methods (fig. 2). In order to compute the correct thrust, the calculated average Mach number should be 1.34 times the effective value. From figure 4, the actual Mach number ratio is less than this value for the mass-derived method, and more for the other methods.

It is shown in the section ANALYSIS that the mass-flow errors indicated in figure 5 can be eliminated by redefining the Mach number and static pressure of the uniform flow. The Mach numbers required to accomplish this are presented in figure 7.

The Mach number errors for both the mass-flow-weighting and area-weighting methods are greatly reduced as compared with the original errors shown in figure 4. In the case of the area-weighting method, the calculated Mach numbers are still larger than the effective values. However, the calculated Mach numbers are now lower than the effective values for the mass-flow-weighting method. These results are consistent with the calculated-to-effective total-pressure ratios shown in figure 3. It can be deduced from equation (14) that the Mach number ratios of figure 7 will be inversely proportional to these total-pressure ratios.

The values of static pressure required to satisfy the mass flow are compared with the effective values determined from the exact weighting procedure in figure 8. The errors in total-pressure calculation (fig. 3) and Mach number calculation (fig. 7) tend to compensate (eq. (15)), so that the static-pressure error is greatly reduced as compared with the measured pressure shown in figure 2. The calculated static pressures for the area-weighting method were less than the effective value. For the mass-flow-weighting method, the calculated pressures exceeded the effective value. These trends arise from the predominant effect of total pressure, as compared with Mach number, in the static-pressure calculation.

The momentums calculated with the static pressures and Mach numbers that satisfied the integrated mass flow are shown in figure 9. In general, these values are less in error than the values computed from the measured static pressure (fig. 6). An exception occurred with the area weighting of the separation profile. In this case, the calculated momentum obtained with the assumption of measured static pressure was slightly greater than the integrated value, whereas that obtained for conditions satisfying the mass flow was less than the integrated value.

The seriousness of the errors introduced by the various weighting methods depends on the use to which the averaged flow quantities are applied. The simple determination of diffuser total-pressure recovery, for example, is only subject to the errors indicated in figure 3. In the usual range of duct Mach number for which such data are evaluated

(Mach numbers less than 0.4), the errors associated with any of the weighting methods are small for the profiles examined. When the averaged quantities are to be utilized in broader applications, however, the errors arising from the various weighting methods may become more significant.

Diffuser characteristics. - The diffuser pressure-recovery - air-flow characteristics that would be predicted by the various weighting methods for the separation profile are indicated in figure 10. In the calculation of this figure, the average static pressure was assumed to correspond to the measured value. It was further assumed that the effective total-pressure recovery was 0.90 at an effective duct Mach number of 0.3, corresponding to critical flow, and was constant in the subcritical flow region of the inlet.[1]

Inasmuch as the mass-derived method of averaging has no mass-flow error, the only difference between the diffuser characteristic predicted by this method and the mass-momentum characteristic occurs in the level of the critical and subcritical pressure recoveries. The mass-flow errors introduced by the mass-flow-weighting and area-weighting methods combine with the total-pressure errors associated with these averaging methods to cause marked shifts in the predicted diffuser characteristic as compared with the mass-momentum characteristic. In the supercritical flow region the corrected air flows predicted by the approximate averaging methods at a given level of pressure recovery are in error in the same proportion as the mass-flow error indicated in figure 4. Conversely, at a given value of corrected air flow, large apparent differences in total-pressure recovery result with the various averaging methods. The choice of averaging method would thus have a large influence on the selection of inlet size to match a desired engine air-flow rate or on the prediction of the operating pressure-recovery level of an engine-inlet combination.

The shift in apparent diffuser characteristic illustrated by figure 10 would be less marked with the other profiles considered in this analysis, inasmuch as the total-pressure and mass-flow errors are smaller than for the separated profile. For weighting methods in which the mass flow is satisfied, the error in diffuser characteristic would be confined to the subcritical pressure-recovery level, regardless of the profile.

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[1]Subcritical flow is defined as the regime where the absolute mass flow varies with changes in discharge pressure. When mass flow is independent of back-pressure changes, the inlet flow is said to be supercritical. This is the hyperbolic region of the curves in figure 10. The intersection of these two flow regimes is termed the critical flow condition.

Diffuser drag calculations. - Figure 11 schematically illustrates the type of research model installation frequently used to evaluate combined internal and external flow problems of engine-inlet installations. Although shown as a nose or nacelle inlet, the same type of installation and support system can be used to study fuselage inlets. The model is supported from a sting by a balance which measures the sum of the thrust and drag forces exerted on the model. Internal air-flow conditions are regulated by a plug in the discharge duct which is supported from the sting. (This plug is generally remotely actuated to vary the air-flow conditions.) Internal flow conditions are evaluated by measurements at a flow measuring station in a region corresponding to the compressor inlet in the model prototype.

Since the duct is cylindrical downstream of the flow measuring station, the only axial force on this section is a small viscous shear force which is generally neglected. The momentum evaluated at the measuring station can therefore be used to determine the thrust force on the model. By subtracting the thrust force from the balance force, the external drag of the model can be determined.

It is shown in equation (B6) of appendix B that errors in momentum or mass-flow calculation at the measuring station cause errors in a drag-coefficient parameter according to the relation

$$\frac{\Delta C_D}{\dfrac{P_e}{P_0}\dfrac{A}{A_{ref}}} = \frac{2}{\gamma M_0^2}\frac{p_0}{P_0}\frac{p}{P_e}\frac{P_e}{p_e}\left[\frac{\varphi}{pA}\left(1 - \frac{\varphi_c}{\varphi}\right) - \gamma\left(\frac{V}{a_a}\right)_0 \frac{m\sqrt{\dfrac{RT}{\gamma}}}{pA}\left(1 - \frac{m_c}{m}\right)\right] \tag{26}$$

The magnitudes of these errors for a free-stream Mach number of 2.0 are indicated in figure 12 for the various profiles and for weighting methods in which the measured static pressure is satisfied. The sign convention is such that positive errors correspond to calculated drag coefficients that are less than the correct values.

With each of the weighting methods, the error increased with increases in the duct Mach number in accordance with the increasing errors in mass-flow and momentum shown in figures 5 and 6. In general, the mass-derived method, in which the mass flow as well as the measured static pressure is satisfied, gave the lowest drag errors.

The importance of the errors indicated in figure 12 depends upon the relative importance of the induction system to the over-all model. If, for example, the model represented by these error curves is a nacelle in which the duct area is 90 percent of the frontal area and the pressure recovery is 0.8, then the absolute error in drag coefficient based on the frontal area would be 72 percent of the indicated parameter error. At Mach number 2.0, the nacelle drag coefficient may be on the order of 0.1 to 0.15 for an effective duct Mach number of 0.2. The indicated errors may thus become a large fraction of the desired value. If, on the other hand, the error curves of figure 12 apply to an inlet mounted on a fuselage in which the duct area is a smaller fraction of the fuselage frontal area, the relative importance of the indicated errors is greatly reduced.

The magnitude of the drag-coefficient errors due to errors in the weighting method is greatly reduced for weighting methods in which the integrated mass flow is satisfied, as shown in figure 13. The error curves for the mass-derived method are reproduced from figure 12. Both the mass-flow-weighting method and the area-weighting method produce less error than the mass-derived method with this criterion. As compared with the method in which the measured static pressure was used (fig. 12), the errors introduced by the mass-flow-weighting method are reduced about 90 percent. For the area-weighting method, the errors with the mass flow satisfied are only on the order of one-fifth the errors when the measured static pressure was used. The sign of the errors obtained from the area weighting method is generally reversed between figures 12 and 13. This corresponds to the shift in value of the calculated momentum relative to the true momentum shown between figures 6 and 9. Except for the separation profile, the lowest drag-coefficient errors are obtained with the area-weighting method when the mass flow is satisfied.

Figures 12 and 13 illustrate possible drag-coefficient errors at a free-stream Mach number of 2.0. The effect of free-stream Mach number is illustrated in figure 14. The profiles evaluated in this figure all have a maximum duct Mach number of 0.4, which corresponds to an effective Mach number of about 0.35 in each case. These calculations are for the weighting methods in which the uniform-flow static pressure is assumed equal to the measured value; hence, the mass-flow errors indicated in figure 5 are included. Similar trends would be observed for the weighting methods in which the integrated mass flow was satisfied. The magnitude of the drag-parameter errors would be decreased in the latter case, however.

The increasing error in drag parameter with increasing supersonic Mach number does not necessarily imply an increase in the absolute drag-coefficient error of the same proportion. The total-pressure-recovery term in the denominator of the drag parameter will generally decrease with increasing Mach number. This will compensate in part for the increase in parameter error. For such cases, the anticipated error in drag coefficient may remain relatively constant throughout the supersonic Mach number range. If, on the other hand, highly efficient inlets are being considered at high Mach numbers, the drag-coefficient error will increase for a given level of flow distortion as compared with the errors resulting at lower Mach numbers.

Inlet pressure recoveries may be expected to remain at a generally high level throughout the subsonic Mach number range. It would be anticipated that the potential error in drag coefficient would therefore increase as Mach number is reduced unless there was a concomitant improvement in the duct profile.

Variable-area-duct calculations. - In many duct flow applications, uniform-flow properties are calculated at a flow measuring station by one of the weighting methods. One-dimensional flow equations are then used to compute flow properties at other stations in the duct by the assumption of appropriate total-pressure losses. These resultant properties are affected by the errors previously demonstrated to be associated with the various weighting methods. Additional errors are introduced if there are area changes in the duct.

The nature of the errors introduced in variable-area-duct calculations can be illustrated by the flow shown in figure 15. It has been assumed in this flow that a uniform static pressure exists at each station and that each filament of the flow expands isentropically between the two stations.

Each filament diffuses to a higher static pressure as the flow passage area increases. The static-pressure rise is constant across all filaments; consequently, the filaments having low initial velocity undergo a greater deceleration than those with high velocity. The expansion rate varies as a result, and the low-velocity filaments occupy a larger fraction of the final duct area than of the initial duct area.

As previously shown, the mass, momentum, and energy of the nonuniform flow at each station can be duplicated by a uniform flow determined by the mass-momentum method. The resultant average total pressure at each station includes the mixing losses that would be incurred if the nonuniform flow were allowed to mix fully in a constant-area section. The magnitude of the mixing losses depends on the velocity differences between fluid filaments in the nonuniform flow. These differences are greater after diffusion than in the initial flow. Thus, the uniform flow satisfying the mass, momentum, and energy of the real flow must undergo an apparent total-pressure loss in the diffusion process, even though the real flow expands isentropically. A final flow calculated from the average initial flow by isentropic one-dimensional equations will therefore be in error.

The magnitude of the errors introduced through the assumption of one-dimensional average flow properties is illustrated in figure 16. For this example, the initial profile was assumed linear with $K = 1.0$ (eq. (19)). The final profiles and duct areas were analytically determined for a range of static-pressure ratio for assumed isentropic expansion of the nonuniform flow by the method outlined in appendix C.

Average flow properties were determined at each station by the conventional weighting methods as well as by the mass-momentum method. The figure presents the ratio between the average weighted properties at each station and those calculated by applying isentropic one-dimensional relations to the initial weighted flow.

As previously indicated, there is an effective loss in total pressure in the expansion process when evaluated by the mass-momentum method. Similar losses are calculated by the area-weighting and mass-derived methods. In addition to the loss in total pressure, the average values of Mach number and the calculated momentum and mass flow are lower at each station in the duct than would be predicted by the one-dimensional calculation.

If the average flow properties at each duct station are determined by the mass-flow-weighting method, the one-dimensional equations may be applied without error. With this weighting method, each filament of the nonuniform flow exerts a weight in the average total-pressure determination that is proportional to its increment of mass flow and total pressure. These quantities remain invariant in the expanded filament; consequently, the calculated average total pressure remains constant.

The error shown in figure 16 for each weighting method is a relative error for the given expansion ratio. It represents the difference between the value of the flow property as computed from one-dimensional relations and the value determined from a weighting of the local flow. The previously discussed inherent error between the weighted flow properties and the integrated flow properties must also be considered before the absolute error associated with the application of one-dimensional relations to variable-area duct flows can be determined.

CONCLUDING REMARKS

It has been shown that conventional weighting methods used to obtain uniform flow representations of nonuniform duct flows can cause large errors in the calculated uniform-flow properties. These errors are predominantly associated with the conventional assumption that the measured static pressure can be used in conjunction with a weighted total pressure to define the uniform flow.

An averaging method has been developed which yields uniform-flow properties that reproduce the mass, momentum, and total energy of the nonuniform flow without error for special cases in which the total temperature and static pressure are constant across the duct. The magnitude of the errors introduced by conventional weighting procedures may often warrant the additional complications required to apply this method.

It has also been shown that nonuniform flows through variable-area duct passages result in changes in average flow properties that are not associated with the real thermodynamic flow path. Consequently, additional errors are introduced into nonuniform duct flow calculations when one-dimensional equations are applied to the averaged flow at one station in order to predict the averaged quantities at another station.

These findings indicate that care should be exercised in the selection of a method of averaging nonuniform duct flows and that calculations based upon the weighted flow should be interpreted with caution.

Lewis Flight Propulsion Laboratory
National Advisory Committee for Aeronautics
Cleveland, Ohio, December 13, 1954

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APPENDIX A

SYMBOLS

The following symbols are used in this report:

A	flow area
A_{ref}	reference area for drag coefficient
a_a	stagnation speed of sound
C_D	drag coefficient, $Drag/q_0 A_{ref}$
F_n	net thrust
K	maximum duct Mach number
M	Mach number
m	mass-flow rate
P	total pressure
p	static pressure
q	dynamic pressure, $q = \frac{1}{2}\rho V^2 = \frac{\gamma}{2}pM^2$
R	gas constant
T	absolute total temperature
V	velocity
W	weight-flow rate
x	fractional distance from wall to duct centerline
γ	ratio of specific heats, 1.4 for air
ρ	mass density
δ	total pressure, corrected to NACA standard sea-level conditions, $P/2116$

θ total temperature, corrected to NACA standard sea-level conditions, $T/519$

φ momentum, $\varphi = mV + Ap = pA(1 + \gamma M^2)$

Subscripts:

c calculated

e effective

i initial station in an expanding duct

is isentropic

0 free stream

CN-3 back

APPENDIX B

CALCULATION OF DRAG ERRORS IN DUCTED-BODY INVESTIGATIONS

The net internal force acting on a ducted body is the difference between the outlet and free-stream momentum. If the model is similar to that shown in figure 11, in which the duct is cylindrical downstream of the force measuring station, the only axial force on this section will be a small viscous shear force. This shear force is generally neglected, and the momentum evaluated at the measuring station is assumed equal to the outlet momentum. The net internal force therefore becomes

$$F_n = \varphi - p_0 A - mV_0 \tag{B1}$$

The absolute error in net thrust arising from errors in the determination of the momentum and mass flow in the duct becomes

$$\Delta F_n = \varphi\left(1 - \frac{\varphi_c}{\varphi}\right) - mV_0\left(1 - \frac{m_c}{m}\right) \tag{B2}$$

where φ and m are the integrated values of momentum and mass flow, respectively, and φ_c and m_c are calculated values based upon inexact averaging methods.

The absolute quantities in the terms on the right side of equation (B2) can be reduced to functions of the equivalent duct Mach number by introducing the measured duct static pressure and the total temperature, which gives

$$\frac{\Delta F_n}{pA} = \frac{\varphi}{pA}\left(1 - \frac{\varphi_c}{\varphi}\right) - \gamma\left(\frac{V}{a_a}\right)_0 \frac{m\sqrt{\frac{RT}{\gamma}}}{pA}\left(1 - \frac{m_c}{m}\right) \tag{B3}$$

Since the balance measures the sum of the thrust and drag forces on the model, the error in calculated external drag will be numerically equal to the error in calculated thrust from equation (B3). The resultant error in drag coefficient based on any arbitrary reference area is

$$\Delta C_D = \frac{\Delta F_n}{q_0 A_{ref}} = \frac{2}{\gamma M_0^2} \frac{A}{A_{ref}} \frac{p}{p_0} \frac{\Delta F_n}{pA} \tag{B4}$$

By using equations (B3) and (B4) and the relation

$$\frac{p}{p_0} = \frac{p}{p_e}\frac{p_e}{P_e}\frac{P_0}{p_0}\frac{P_e}{P_0} \tag{B5}$$

the following drag-coefficient-error parameter can be determined, which is a function of free-stream and measuring-station flow conditions only:

$$\frac{\Delta C_D}{\frac{P_e}{P_0}\frac{A}{A_{ref}}} = \frac{2}{\gamma M_0^2}\frac{P_0}{p_0}\frac{p}{p_e}\frac{p_e}{P_e}\left[\frac{\varphi}{pA}\left(1 - \frac{\varphi_c}{\varphi}\right) - \gamma\left(\frac{V}{a_a}\right)_0 \frac{m\sqrt{\frac{RT}{\gamma}}}{pA}\left(1 - \frac{m_c}{m}\right)\right] \tag{B6}$$

APPENDIX C

DETERMINATION OF NONUNIFORM-FLOW PROFILE AFTER ISENTROPIC DIFFUSION

The continuity equation may be written in differential form as

$$dm = \sqrt{\frac{\gamma}{RT}}\, pM\left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{1}{2}} dA \tag{C1}$$

Thus,

$$dA = \frac{M_i\left(1 + \frac{\gamma-1}{2} M_i^2\right)^{\frac{1}{2}} dA_i}{\frac{p}{p_i} M\left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{1}{2}}} \tag{C2}$$

where the subscript i refers to the initial duct station before diffusion.

For isentropic flow,

$$\frac{p}{p_i} = \frac{p}{P}\,\frac{P}{p_i} = \left(\frac{1 + \frac{\gamma-1}{2} M_i^2}{1 + \frac{\gamma-1}{2} M^2}\right)^{\frac{\gamma}{\gamma-1}} \tag{C3}$$

Combining equations (C2) and (C3) gives

$$dA = \frac{M_i\, dA_i}{\left(\frac{p}{p_i}\right)^{\frac{1}{\gamma}}\left\{M_i^2 + \frac{2}{\gamma-1}\left[1 - \left(\frac{p}{p_i}\right)^{\frac{\gamma-1}{\gamma}}\right]\right\}^{1/2}} \tag{C4}$$

For two-dimensional flow, $dA_i/A_i = dx_i$. The required flow area after diffusion therefore becomes

$$A = \frac{A_i}{\left(\frac{p}{p_i}\right)^{\frac{1}{\gamma}}} \int_0^1 \frac{M_i \, dx_i}{\left\{M_i^2 + \frac{2}{\gamma-1}\left[1 - \left(\frac{p}{p_i}\right)^{\frac{\gamma-1}{\gamma}}\right]\right\}^{1/2}} \tag{C5}$$

The Mach number of any filament after diffusion is, from equation (C3),

$$M = \left\{\frac{2}{\gamma-1}\left[\frac{1 + \frac{\gamma-1}{2} M_i^2}{\left(\frac{p}{p_i}\right)^{\frac{\gamma-1}{\gamma}}} - 1\right]\right\}^{\frac{1}{2}} \tag{C6}$$

The coordinate x of the filament after diffusion is

$$x = \frac{\displaystyle\int_0^{x_i} \frac{M_i \, dx_i}{\left\{M_i^2 + \frac{2}{\gamma-1}\left[1 - \left(\frac{p}{p_i}\right)^{\frac{\gamma-1}{\gamma}}\right]\right\}^{1/2}}}{\displaystyle\int_0^1 \frac{M_i \, dx_i}{\left\{M_i^2 + \frac{2}{\gamma-1}\left[1 - \left(\frac{p}{p_i}\right)^{\frac{\gamma-1}{\gamma}}\right]\right\}^{1/2}}}$$

In the example considered herein, the initial profile was assumed to follow the linear equation $M_i = 0.2x_i + 0.8$. From equation (C5) the required flow area after diffusion becomes

$$A = \frac{A_i}{\left(\frac{p}{p_i}\right)^{\frac{1}{\gamma}}}\left\{\sqrt{1 + \frac{2}{\gamma-1}\left[1 - \left(\frac{p}{p_i}\right)^{\frac{\gamma-1}{\gamma}}\right]} - \sqrt{0.64 + \frac{2}{\gamma-1}\left[1 - \left(\frac{p}{p_i}\right)^{\frac{\gamma-1}{\gamma}}\right]}\right\} \tag{C8}$$

The flow coordinate for a given filament of the flow becomes

$$x = \frac{\sqrt{M_1^2 + \frac{2}{\gamma-1}\left[1 - \left(\frac{p}{p_i}\right)^{\frac{\gamma-1}{\gamma}}\right]} - \sqrt{0.64 + \frac{2}{\gamma-1}\left[1 - \left(\frac{p}{p_i}\right)^{\frac{\gamma-1}{\gamma}}\right]}}{\sqrt{1 + \frac{2}{\gamma-1}\left[1 - \left(\frac{p}{p_i}\right)^{\frac{\gamma-1}{\gamma}}\right]} - \sqrt{0.64 + \frac{2}{\gamma-1}\left[1 - \left(\frac{p}{p_i}\right)^{\frac{\gamma-1}{\gamma}}\right]}} \tag{C9}$$

Upon substitution for M_1 from equation (C3), equation (C9) may be simplified to

$$M = \frac{\sqrt{1 + \frac{2}{\gamma-1}\left[1 - \left(\frac{p}{p_i}\right)^{\frac{\gamma-1}{\gamma}}\right]} - \sqrt{0.64 + \frac{2}{\gamma-1}\left[1 - \left(\frac{p}{p_i}\right)^{\frac{\gamma-1}{\gamma}}\right]}}{\left(\frac{p}{p_i}\right)^{\frac{\gamma-1}{2\gamma}}} x + \frac{\sqrt{0.64 + \frac{2}{\gamma-1}\left[1 - \left(\frac{p}{p_i}\right)^{\frac{\gamma-1}{\gamma}}\right]}}{\left(\frac{p}{p_i}\right)^{\frac{\gamma-1}{2\gamma}}} \tag{C10}$$

The profile after diffusion is therefore also linear, and the weighting equations may be solved directly for the uniform-flow properties.

REFERENCE

1. McLafferty, G. H.: A Generalized Approach to the Definition of Average Flow Quantities in Nonuniform Streams. Rep. No. R-13534-9, Res. Dept., United Aircraft Corp., July 20, 1954.

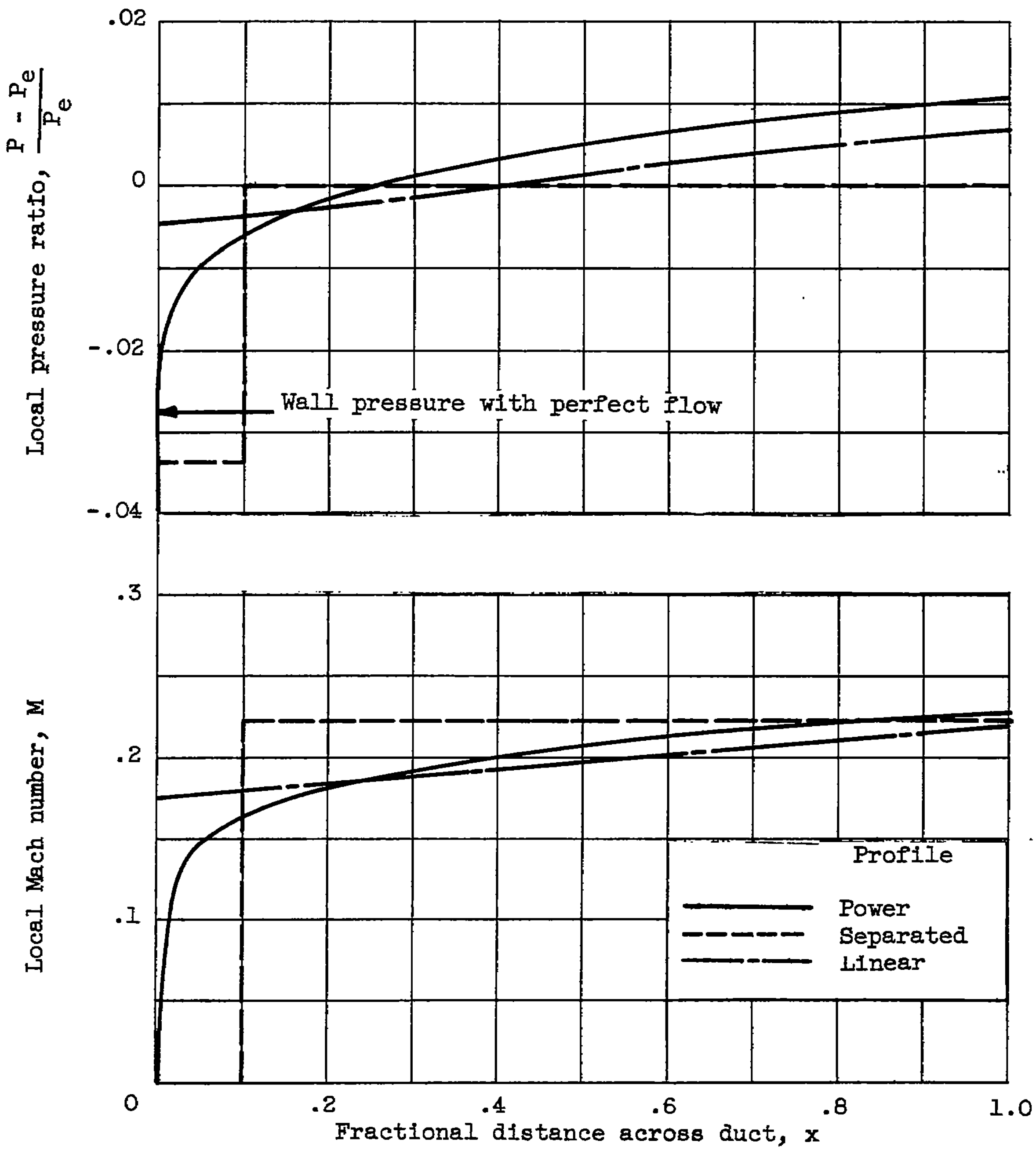


Figure 1. - Typical profiles considered in analysis. Effective duct Mach number, 0.2.

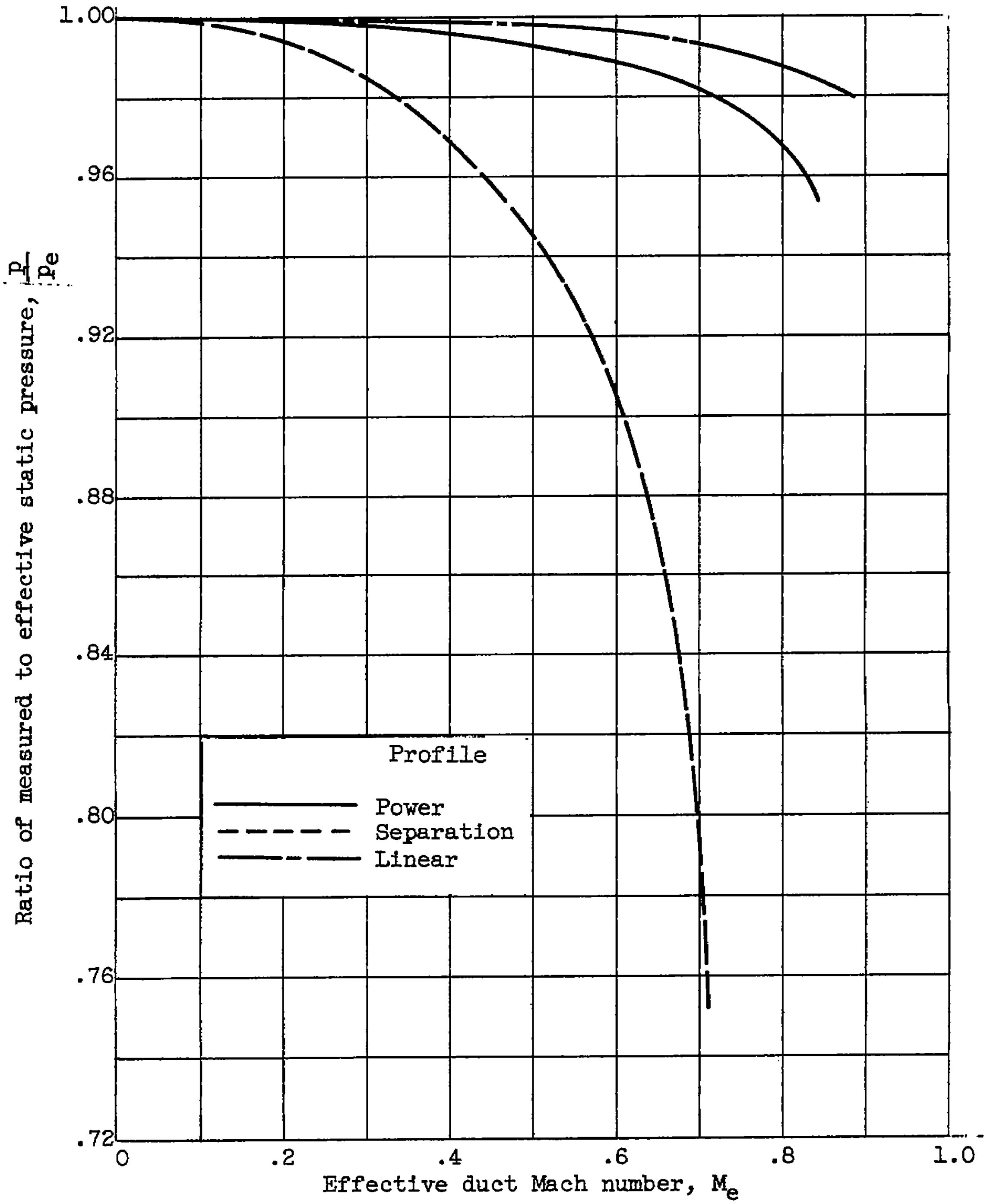
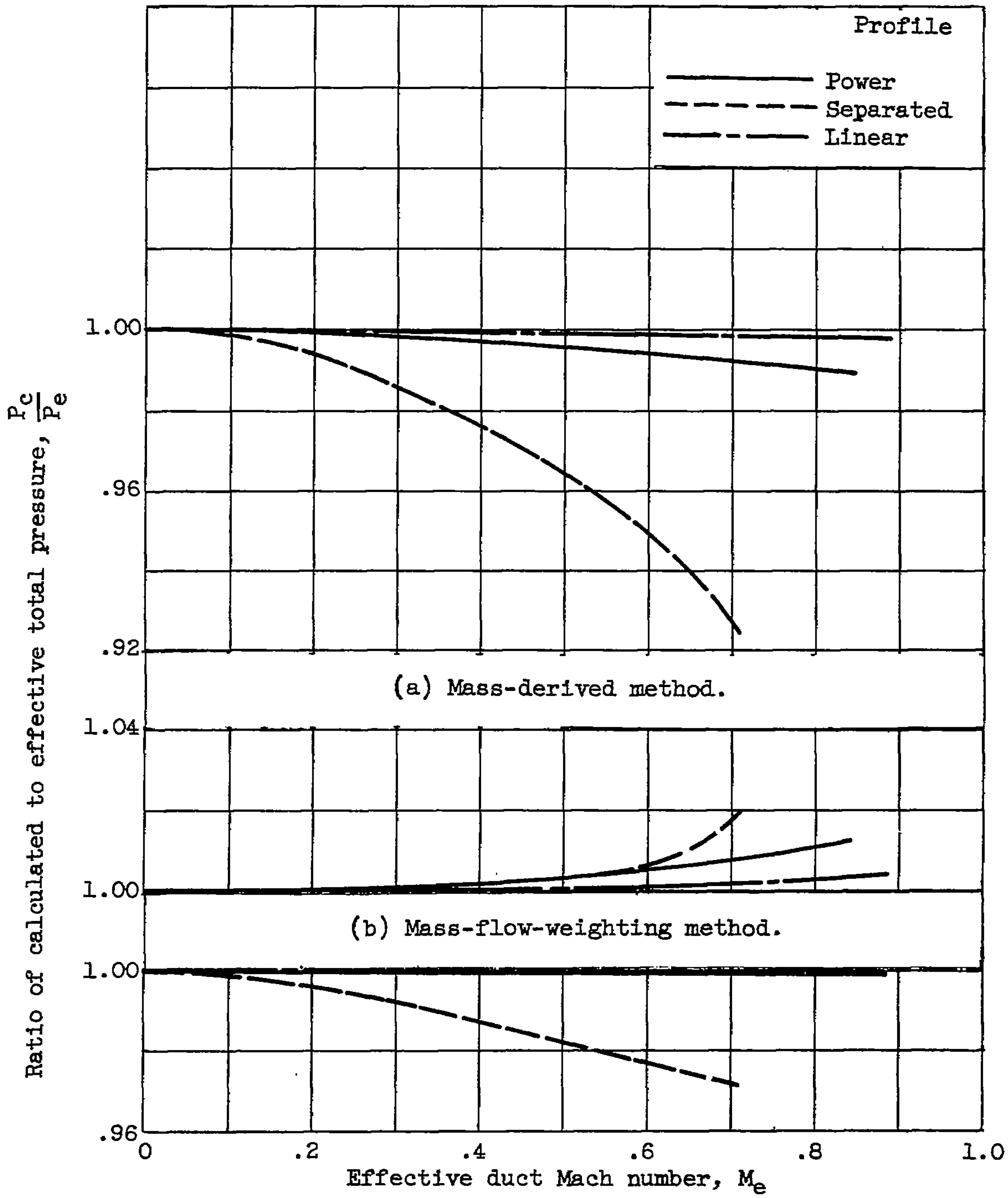


Figure 2. - Ratio of measured to effective static pressure for assumed profiles.



(c) Area-weighting method.

Figure 3. - Total pressures calculated by three weighting methods.

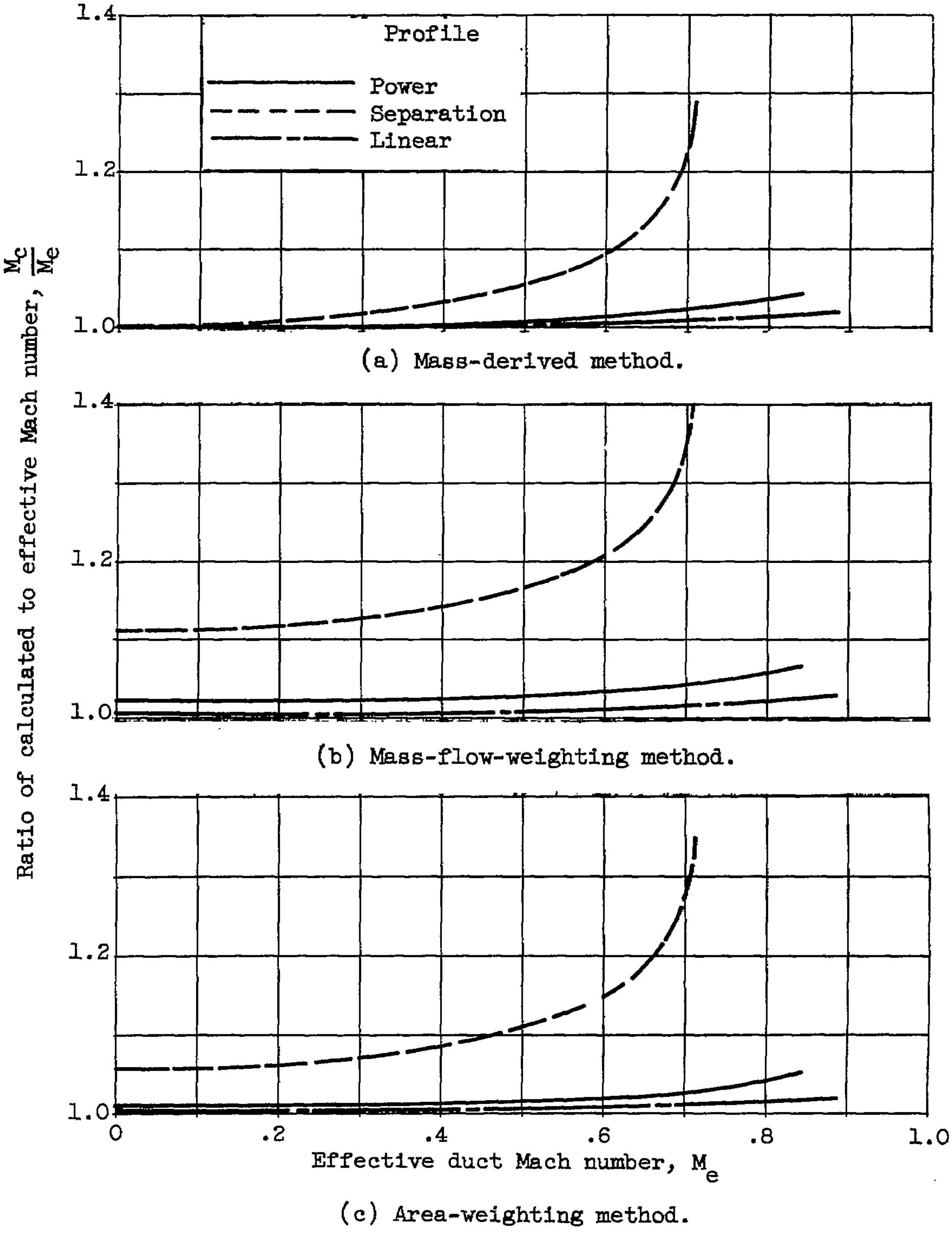


Figure 4. - Duct Mach numbers calculated by three weighting methods. Static pressure equal to measured value.

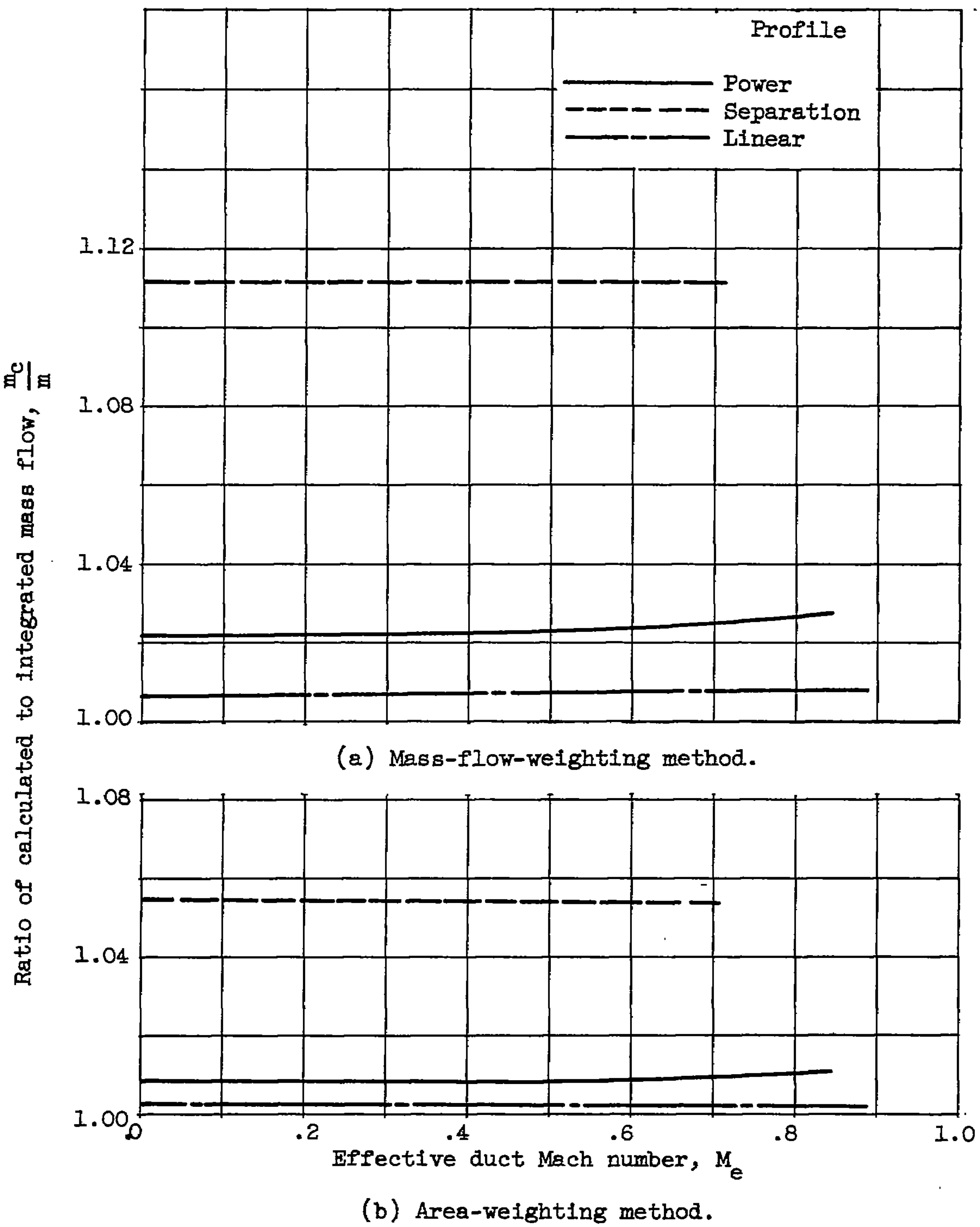
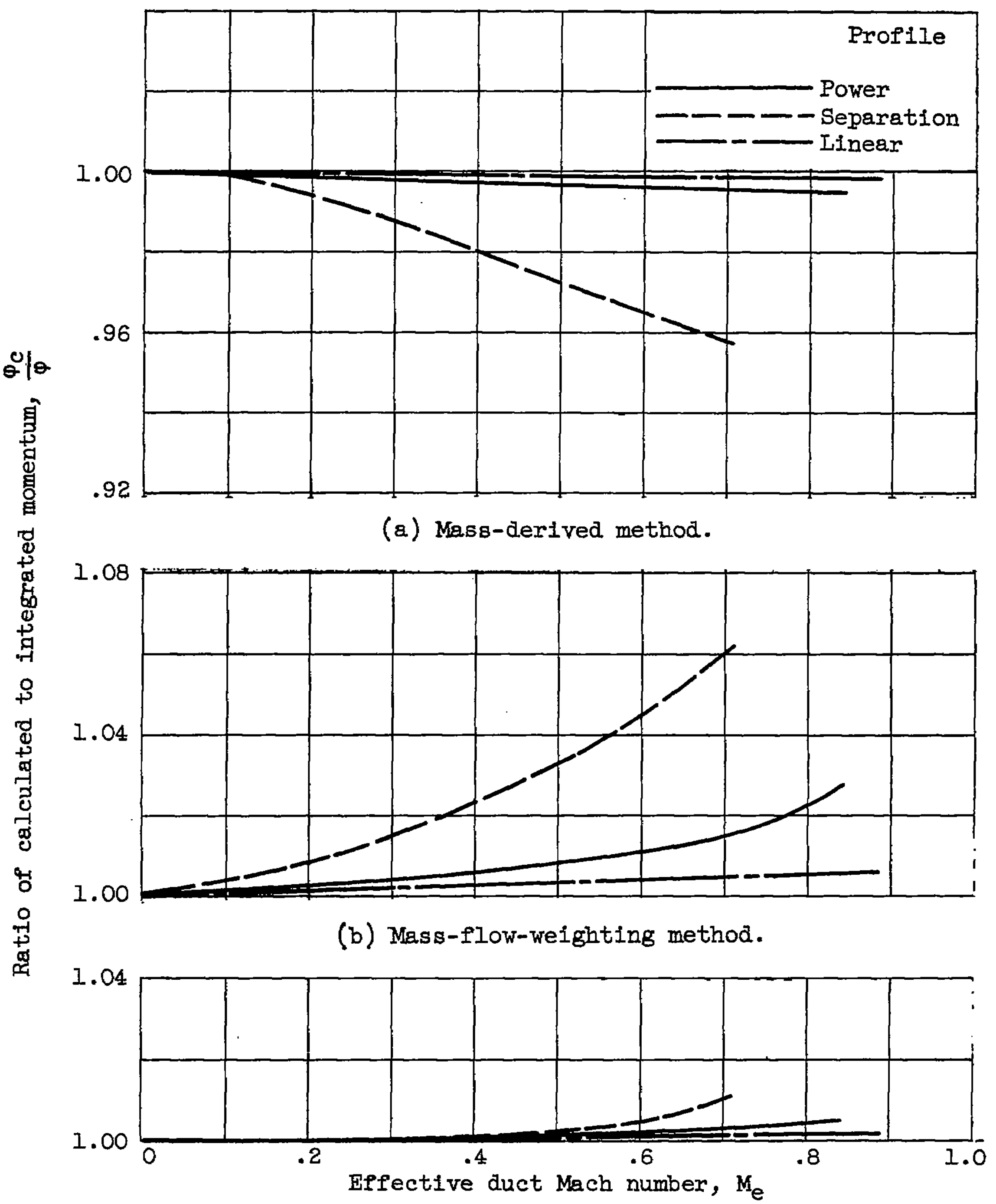
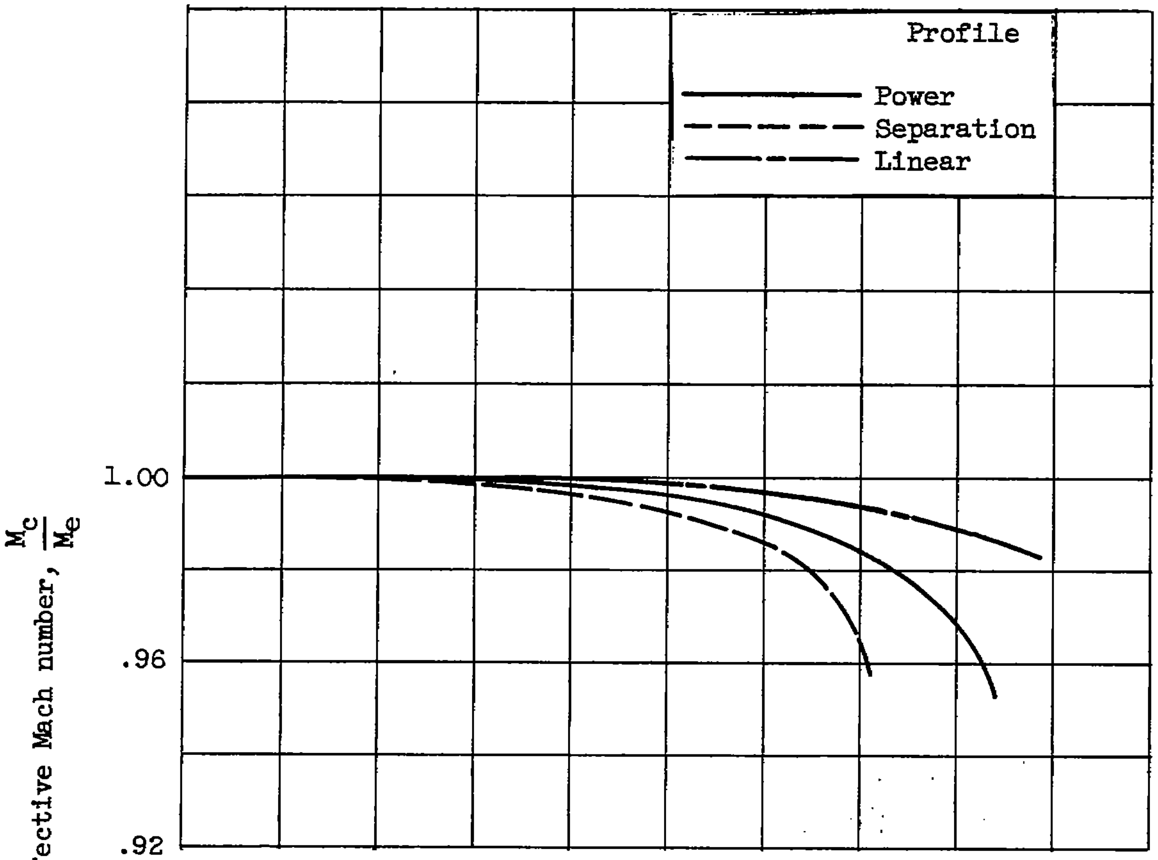


Figure 5. - Mass flow calculated by two weighting methods. Static pressure equal to measured value.

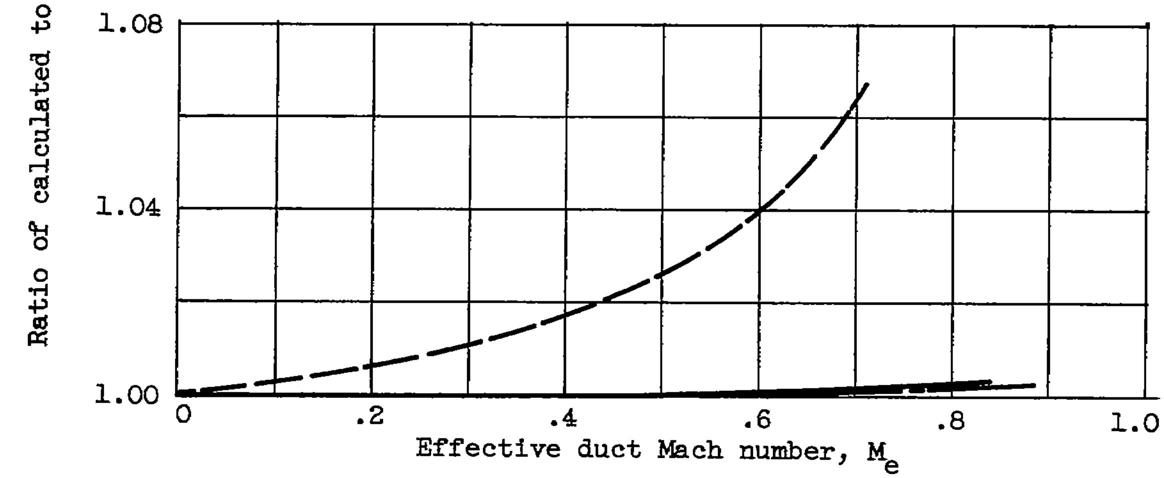


(c) Area-weighting method.

Figure 6. - Momentum calculated by three weighting metho[illegible] Static pressure equal to measured value.

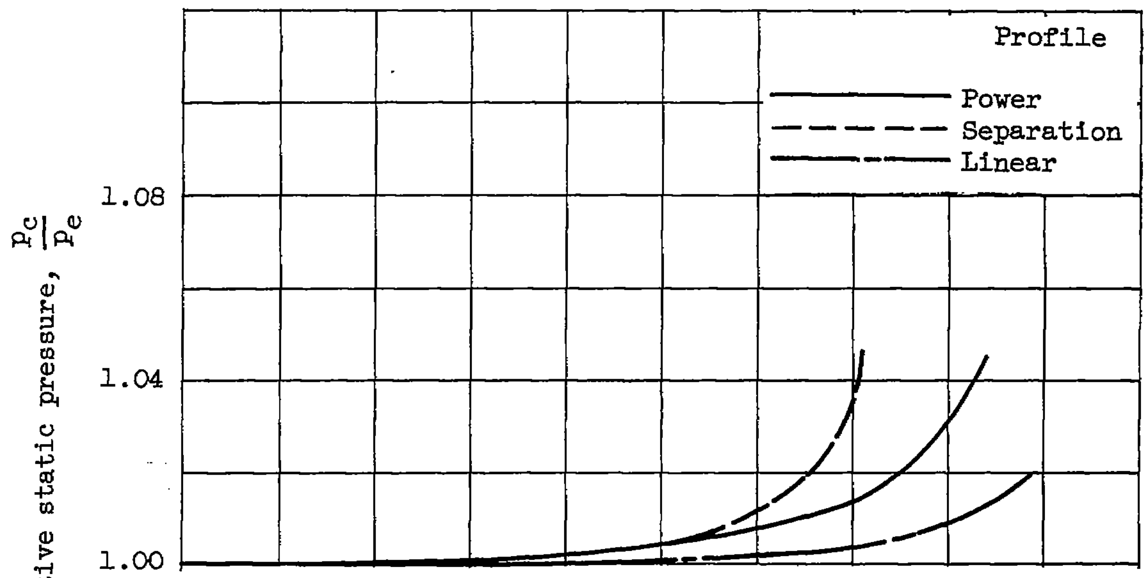


(a) Mass-flow-weighting method.

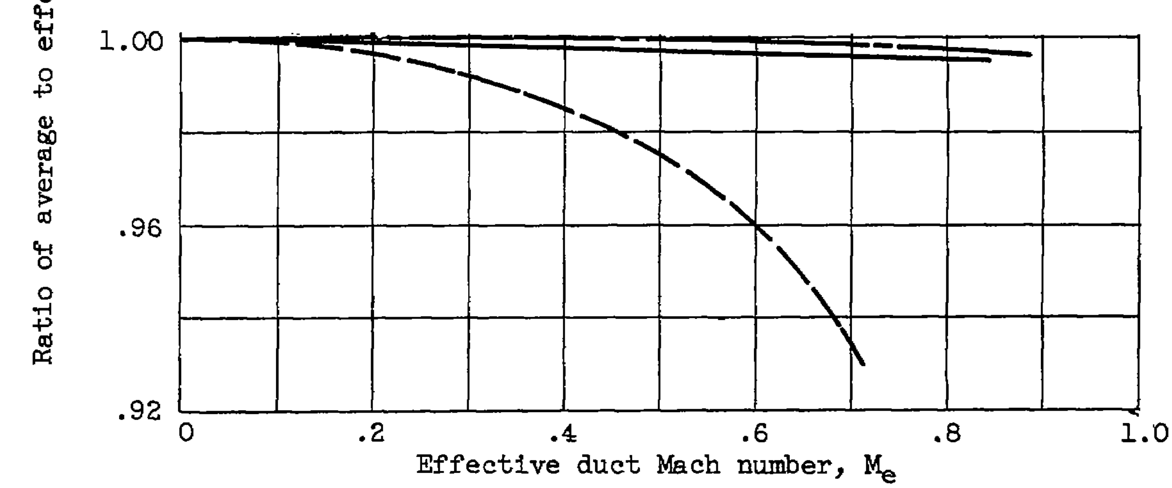


(b) Area-weighting method.

Figure 7. - Duct Mach numbers calculated by two weighting methods; integrated mass flows satisfied.

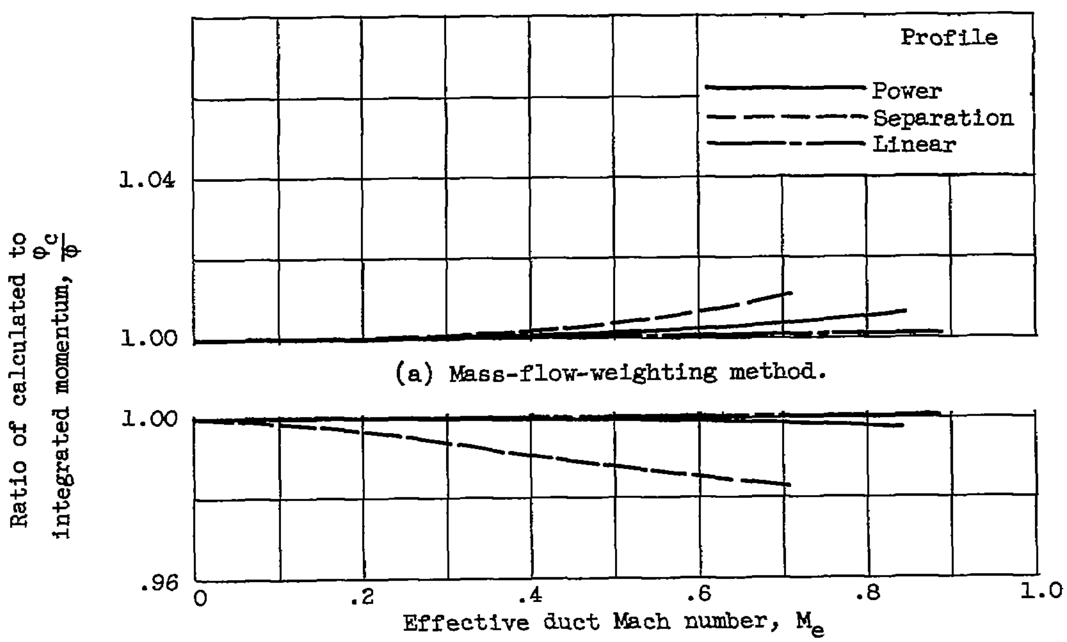


(a) Mass-flow-weighting method.



(b) Area-weighting method.

Figure 8. - Static pressures calculated by two weighting methods to satisfy integrated mass flow.



(b) Area-weighting method.

Figure 9. - Momentum calculated by two weighting methods; integrated mass flows satisfied.

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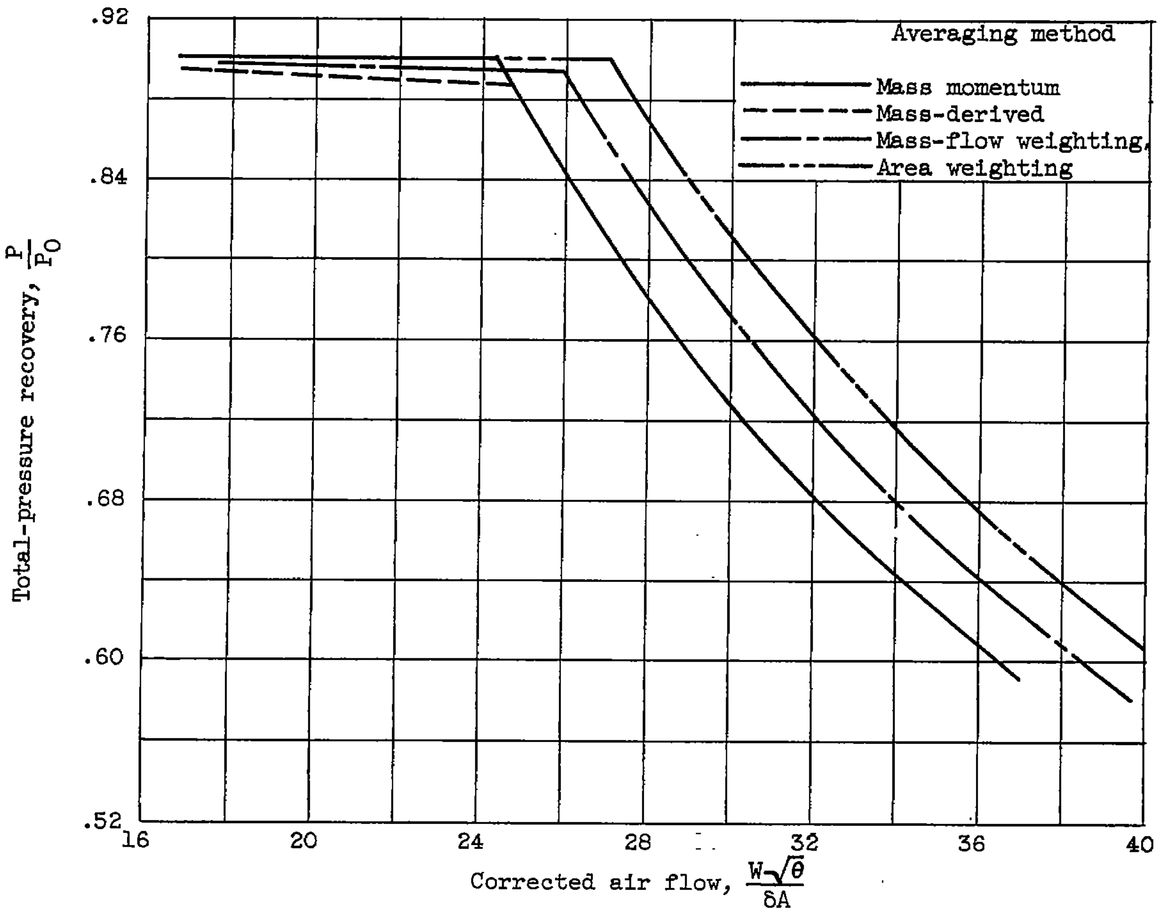


Figure 10. - Comparison of diffuser characteristics predicted by various weighting methods for a separation profile. Static pressure equal to measured value.

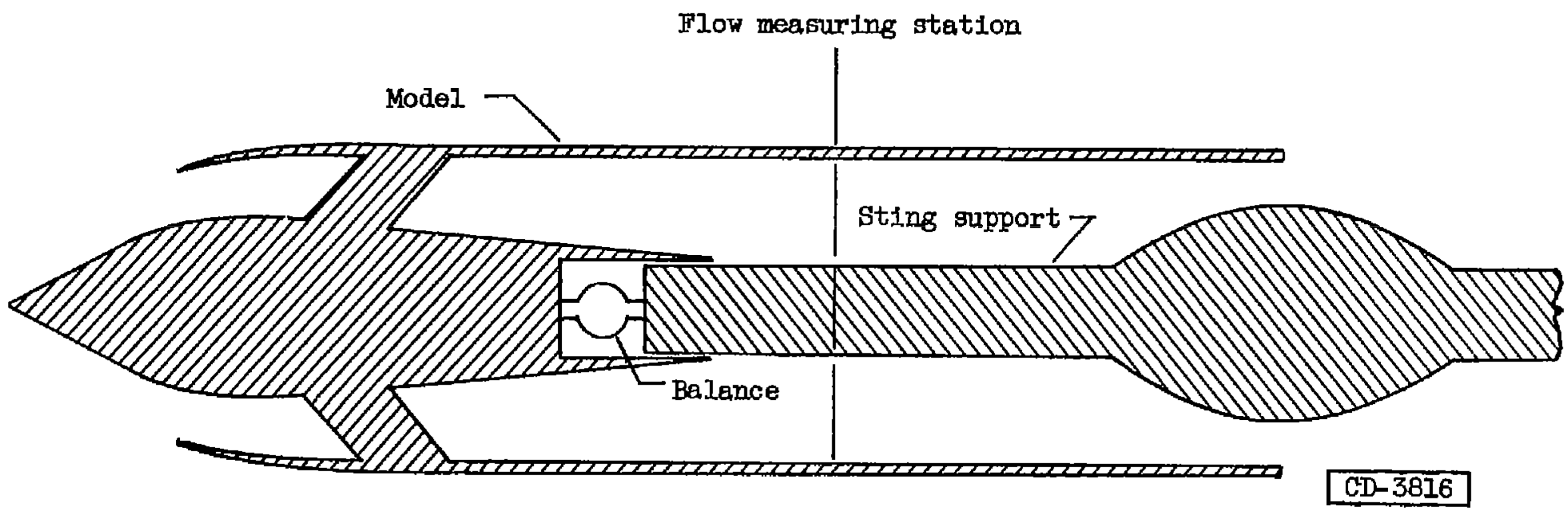
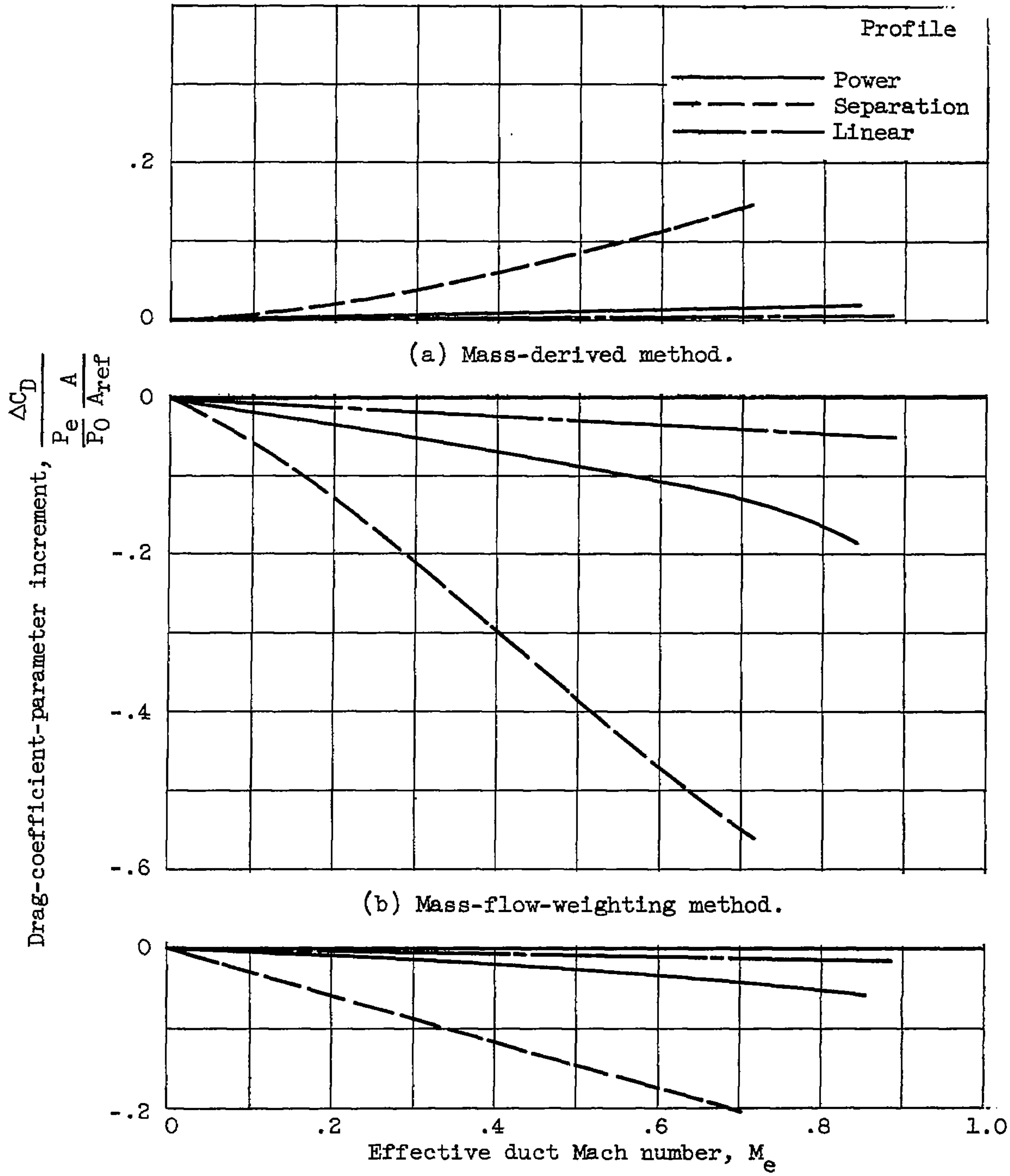


Figure 11. - Schematic representation of typical model used to study combined internal and external flow problems.



(c) Area-weighting method.

Figure 12. - Drag-coefficient errors introduced by various weighting methods. Static pressure equal to measured value; free-stream Mach number, 2.0.

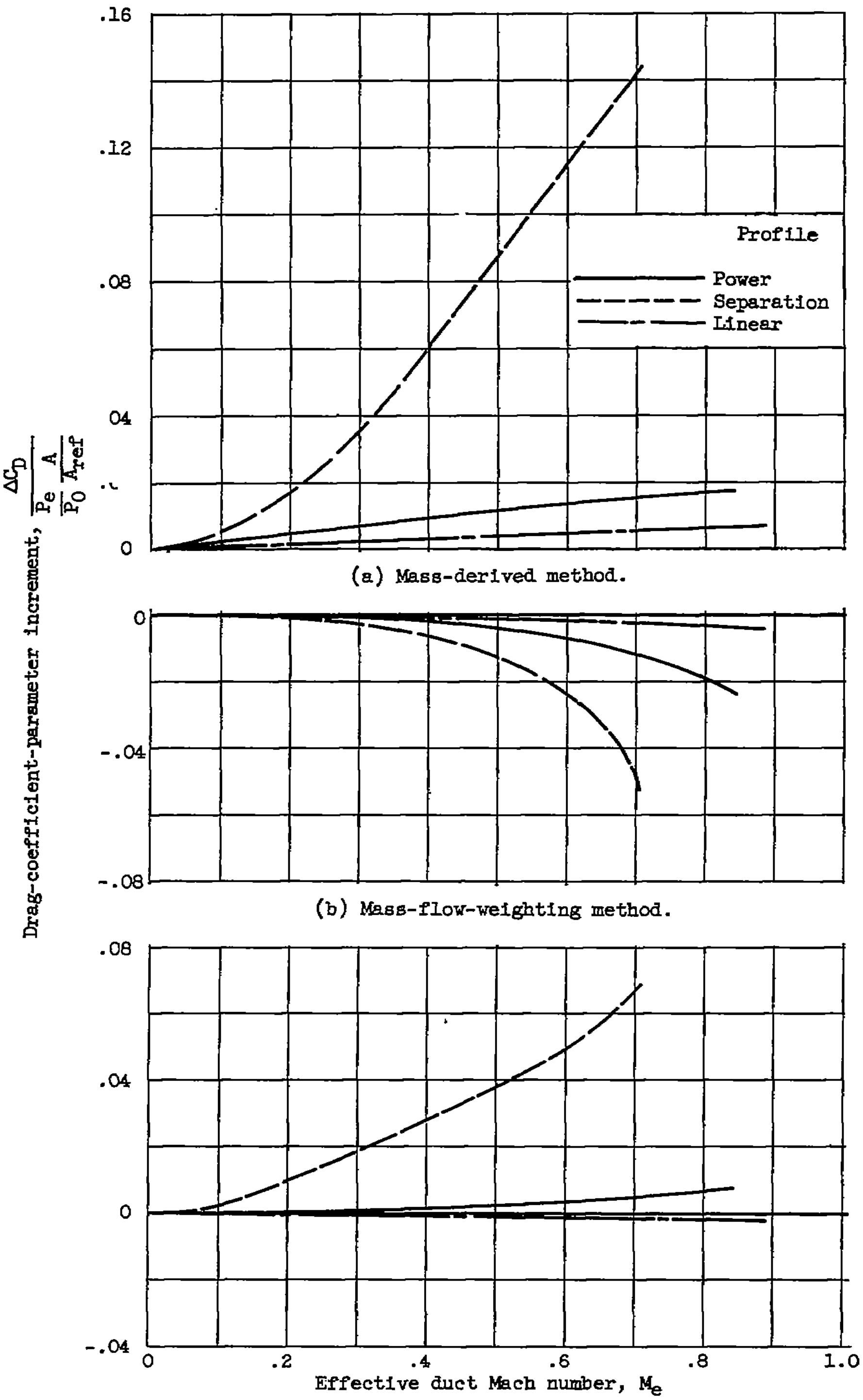
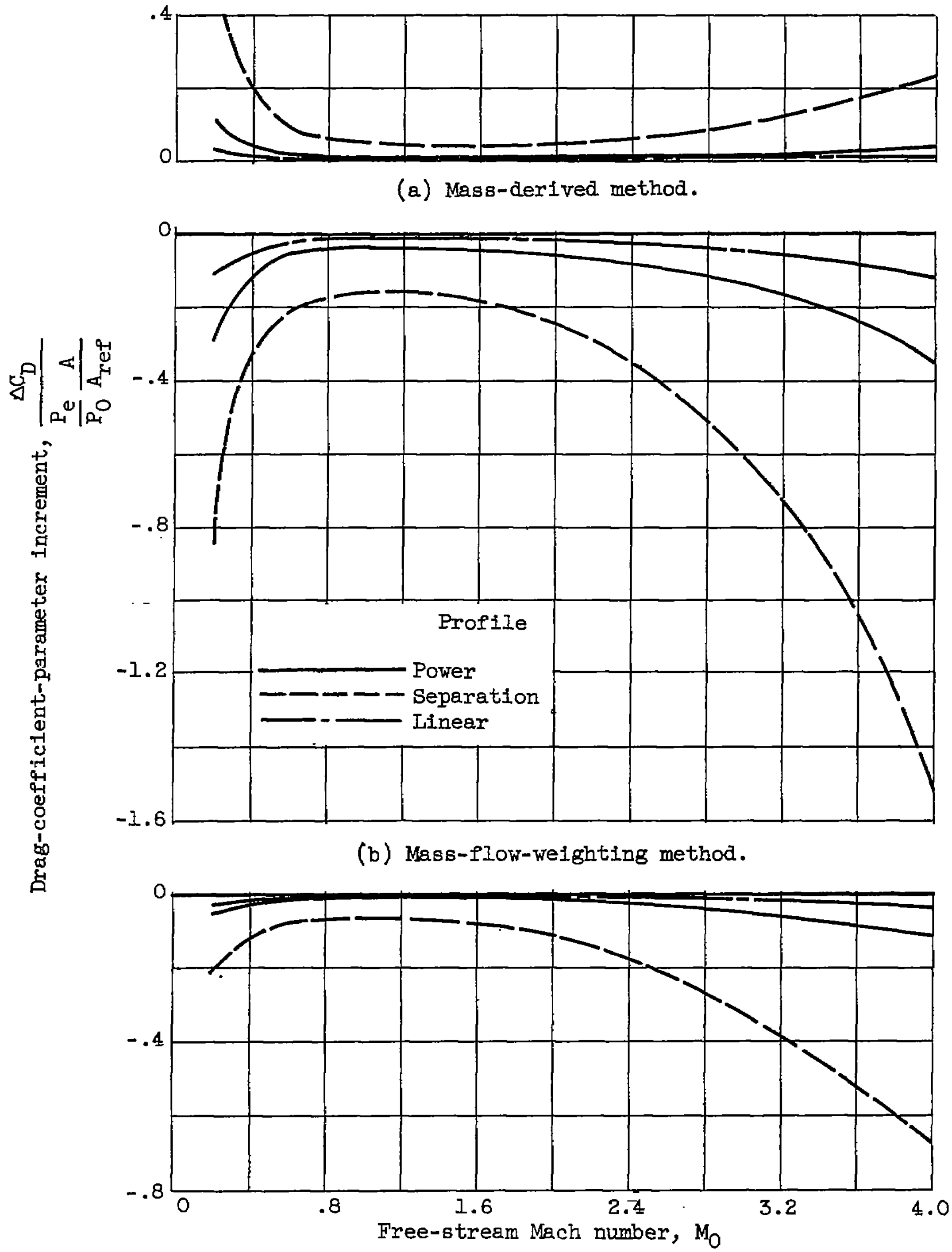


Figure 13. - Drag-coefficient errors introduced by various weighting methods. Integrated mass flows satisfied; free-stream Mach number, 2.0.



(c) Area-weighting method.

Figure 14. - Variation of drag-coefficient error with Mach number. Static pressure equal to measured value; maximum duct Mach number, 0.4 (effective duct Mach number, approx. 0.35).

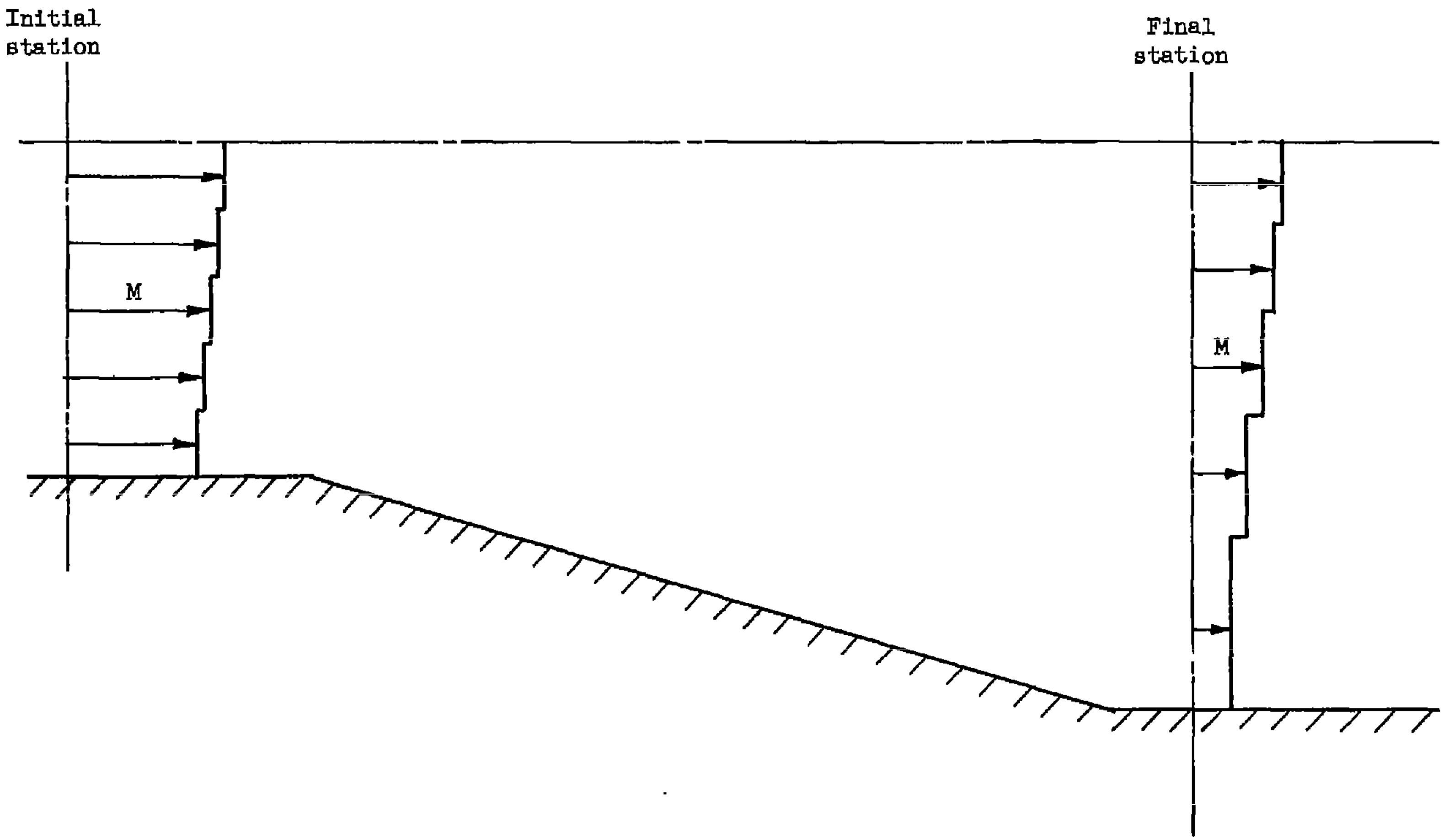


Figure 15. - Typical profile distortion in an isentropic diffusion process.

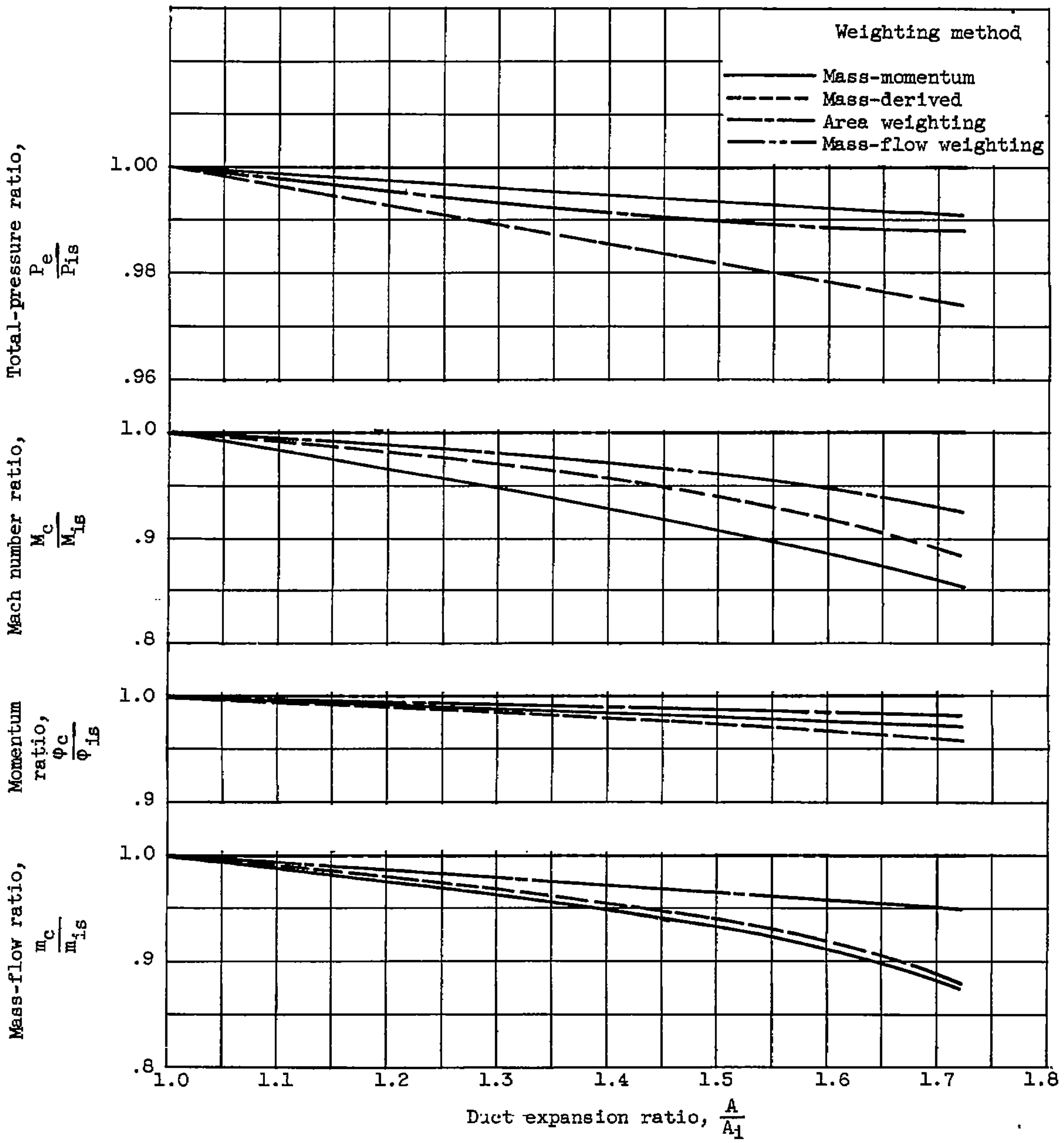


Figure 16. - Comparison of locally weighted and isentropically calculated flow properties in an expanding duct.